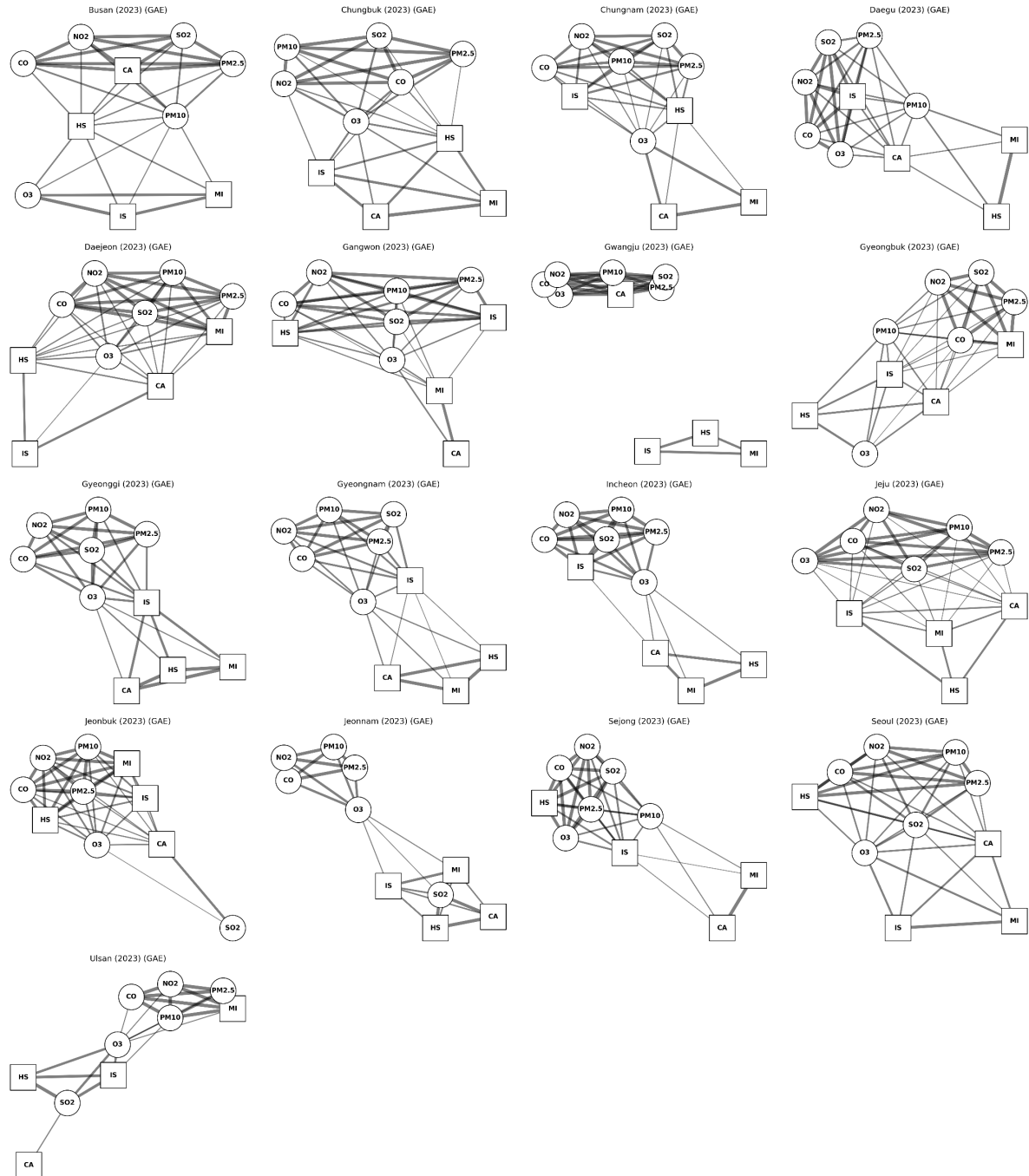


## Supplement 11. Monthly Graph Autoencoder (GAE) Networks Depicting Lagged Pollutant–Disease Associations in 2023



This figure displays Graph Autoencoder (GAE) network visualizations learned separately for each of the 17 administrative regions in South Korea during 2023 using lagged associations ( $t-1$  month pollutant concentrations with  $t$  month emergency room visits).

### Graph Elements:

**Nodes:** Circles represent air pollutants ( $\text{SO}_2$ ,  $\text{NO}_2$ ,  $\text{O}_3$ ,  $\text{CO}$ ,  $\text{PM}_{10}$ ,  $\text{PM}_{2.5}$ ), and squares represent disease categories: cardiac arrest (CA), myocardial infarction (MI), ischemic stroke (IS), and hemorrhagic stroke (HS).

**Edges:** Black lines indicate predicted structural similarity ( $\text{adj\_pred}$ ) between nodes based on lagged temporal relationships. Thicker edges represent stronger connections as inferred by the model.

**Interpretation:**

The networks illustrate how lagged pollutant–disease structural relationships vary by region, with previous month's pollutant concentrations associated with current month's disease outcomes.

Urban areas (e.g., Busan, Incheon, Ulsan) tend to show denser and more interconnected graphs even in lagged analysis, suggesting that metropolitan regions experience more persistent pollutant–disease associations across monthly boundaries.

Non-metropolitan regions (e.g., Gangwon, Chungbuk) often display sparser lagged networks, with fewer strong edges connecting previous month's pollutants to current month's disease nodes, potentially indicating less carryover effects in rural areas.

Overall, these regional GAE networks with lagged associations highlight spatial heterogeneity in the temporal persistence of relationships between air pollution and cardiovascular/cerebrovascular disease burden in 2023.